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### A Simple Method of Determining the Pitch of a Chiral Nematic Liquid Crystal

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## A Simple Method of Determining the Pitch of a Chiral Nematic Liquid Crystal

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*The pitch value  $P$  of a chiral nematic liquid crystal is a critical parameter for azimuthal anchoring strength measurement. The Grandjean-Cano wedge method is a conventional technique for pitch determination, but the perfect wedge cell with known angle and uniform disclination lines are required. In this work, we have developed an improved Grandjean-Cano method, which is simple and gives accurate pitch value. The sensitivity of this new method is  $\pm\lambda/2\Delta n$ .*

**Keywords:** azimuthal anchoring strength; chiral nematic liquid crystal; pitch

### I. INTRODUCTION

In the development of new alignment methods, the azimuthal anchoring strength has been a critical index that helps us to characterize the alignment method quantitatively. An anti-parallel cell filled with a chiral nematic liquid crystal is commonly used in azimuthal anchoring measurement [1]. Accurately determining the pitch value of the chiral nematic then is important for obtaining accurate azimuthal anchoring strength.

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The conventional method to determine the pitch is the Grandjean-Cano wedge method [2]. In this method, a wedge shape cell filled with the chiral nematic liquid crystal is used. Domains with twisting angles being multiples of  $\pi$  are formed and the disclination lines at the boundaries of the domains can be seen either by bare eyes or microscope. With a known wedge angle, the pitch can be calculated by measuring the spacing between the disclination lines. However, the wedge is not perfect in reality, the angles and the spacing between disclination lines are not uniform due to bending or stress on glass substrate. An uncertainty as large as 10% is common.

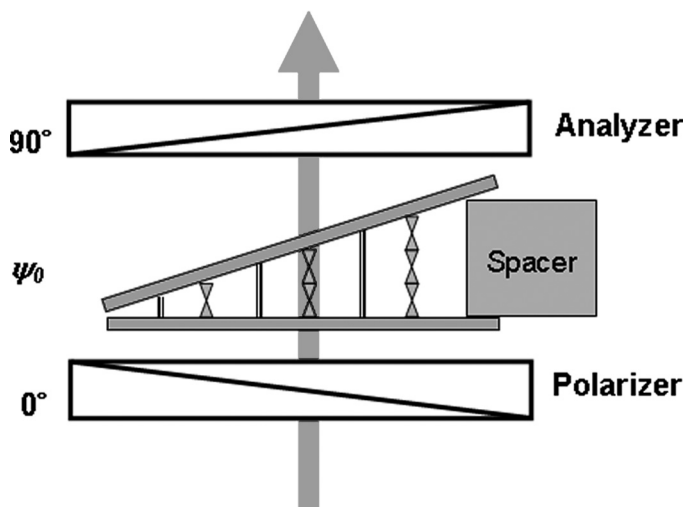
In this work, we demonstrate a simple and accurate method to measure the pitch value. In section II, we describe the sample requirement and the measuring method. The theory and simulation of this method are given in section III. We give detailed discussions about this method and its improvement for azimuthal anchoring strength measuring in section IV. Brief conclusions are given in the last section.

## II. METHODS

The sample we use to determine the pitch is the same as the one used in the Grandjean-Cano method. The cell is constructed using two indium tin oxide (ITO) glass substrate, the surfaces of which are coated with polyimide SE-130B (from Nissan Co.) and rubbed with nylon fabric. The rubbing directions of the two substrates are anti-parallel to each other. The wedged cell is formed with a piece of Mylar space at one end and the filled with nematic liquid crystal E7 (from Merck Co.) that is doped with chiral material ZLI-811 (from Merck Co.). A pair of crossed polarizers (the polarizer and the analyzer) is used and the cell is put in between with the rubbing direction having an angle  $\psi_0$  from the polarizer transmission axis. By using an extended white light source such as the common film viewer and a narrow color filter ( $\lambda = 546\text{nm}$ ), the sample between crossed polarizers shows many bright and dark fringes between two adjacent disclination lines. Both of the fringes and disclination lines can be observed with bare eyes. The setup and the sample are shown in Figure 1. The photographs of three samples using different spacers are shown in Figure 2.

## III. THEORY & SIMULATION

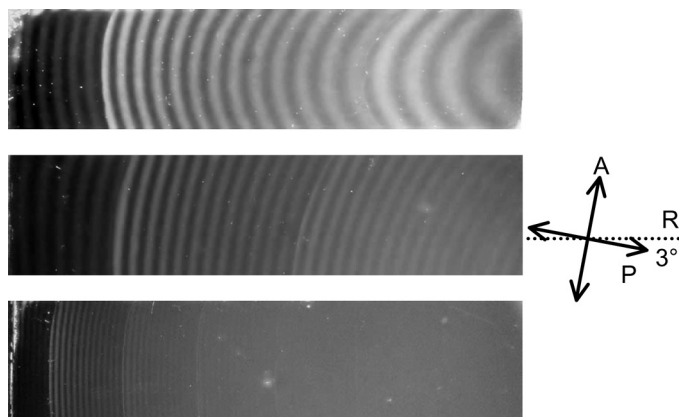
Saitoh and Lien proposed a method based on the measurement of actual twist angle by using Jones matrix equation [3]. For a uniformly



**FIGURE 1** The setup of this new method. The incident light is monochromatic.

twisted nematic cell, the optical transmittance  $T$  can be written as

$$T = \left[ \frac{1}{\sqrt{1+u^2}} \sin(\sqrt{1+u^2}\theta) \sin(\theta - \psi_{pol}) + \cos(\sqrt{1+u^2}\theta) \cos(\theta - \psi_{pol}) \right]^2 + \frac{u^2}{1+u^2} \sin^2(\sqrt{1+u^2}\theta) \cos^2(\theta + 2\psi_0 - \psi_{pol}), \quad (1)$$



**FIGURE 2** Photographs of the wedged samples with different spacers at the wider end: (a) 50  $\mu\text{m}$  (b) 75  $\mu\text{m}$  (c) 250  $\mu\text{m}$ . P: polarizer axis, A: analyzer axis, and R: rubbing direction.

where  $\theta$  is the twist angle of liquid crystal,  $\psi_{pol}$  is the angle between the transmission axes of the analyzer and the polarizer,  $u = \pi\Delta n d / \lambda \theta$  with  $\Delta n$  being the birefringence of chiral nematic liquid crystal and  $\lambda$  is the wavelength of the light and  $d$  the thickness of liquid crystal layer.

In this method,  $\psi_{pol}$  is fixed as  $90^\circ$  and  $u$  can be replaced by  $\Gamma/2\theta$ , where the phase retardation  $\Gamma$  of the wedge sample  $2\pi\Delta n d / \lambda$ . The transmittance  $T$  is replaced by

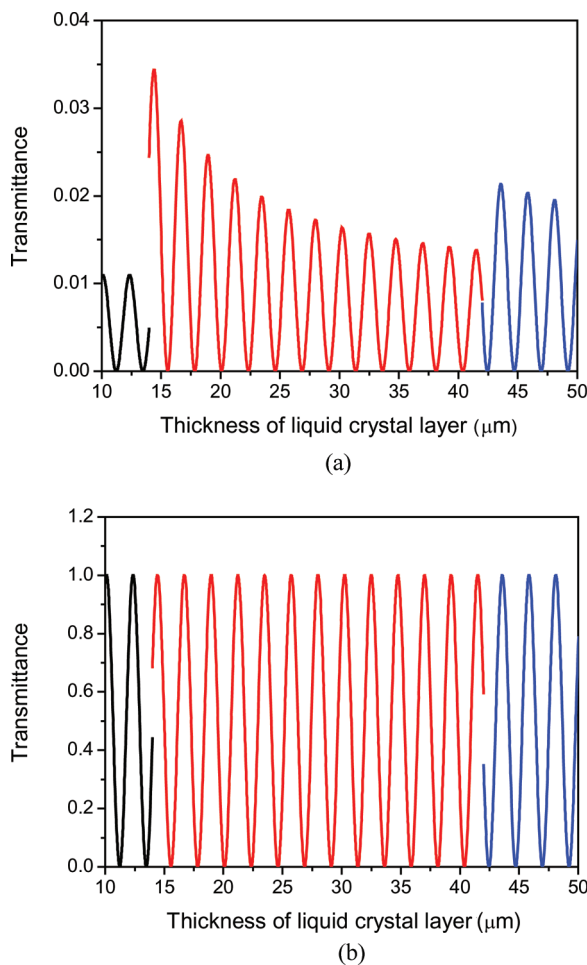
$$T = \frac{1}{\Gamma^2 + 4\theta^2} \left\{ \left[ \sqrt{\Gamma^2 + 4\theta^2} \cos\left(\frac{\sqrt{\Gamma^2 + 4\theta^2}}{2}\right) \sin(\theta) - 2\theta \cos(\theta) \sin\left(\frac{\sqrt{\Gamma^2 + 4\theta^2}}{2}\right) \right]^2 + \Gamma^2 \sin^2\left(\frac{\sqrt{\Gamma^2 + 4\theta^2}}{2}\right) \sin^2(\theta - 2\psi_0) \right\} \quad (2)$$

within each domain in the wedged cell the twisted angle is fixed, however, the cell gap changes monotonically with position, the phase retardation  $\Gamma$  also changes accordingly with position and the fringe patterns are then resulted in each domain.

Grandjean realized that cholesteric liquid crystal prepared under planar alignment conditions in a wedged cell exhibits disclination lines whenever the twist angle changes by  $\pi$  [2]. When it is between  $N$ -th and  $(N+1)$ -th disclination lines, the twist angle  $\theta$  is  $N\pi$ , where  $N = 0, 1, 2$ , etc. By substituting  $\sin(\theta) = 0, \cos^2(\theta) = 1, \sin^2(\theta - 2\psi_0) = \sin^2(2\psi_0)$  and  $\sin^2\left(\frac{\sqrt{\Gamma^2 + 4\theta^2}}{2}\right) = \frac{1}{2}\left(1 - \cos\left(\sqrt{\Gamma^2 + 4\theta^2}\right)\right)$  into Eq. (2), the optical transmittance equation can be further simplified as

$$T = \frac{1}{\Gamma^2 + 4\theta^2} \left[ 4\theta^2 \sin^2\left(\frac{\sqrt{\Gamma^2 + 4\theta^2}}{2}\right) + \Gamma^2 \sin^2\left(\frac{\sqrt{\Gamma^2 + 4\theta^2}}{2}\right) \sin^2(2\psi_0) \right] = \frac{1}{2} \left( \frac{\Gamma^2 \sin^2(2\psi_0) + 4\theta^2}{\Gamma^2 + 4\theta^2} \right) \left( 1 - \cos\left(\sqrt{\Gamma^2 + 4\theta^2}\right) \right) \quad (3)$$

For the domain between  $N$ -th and  $(N+1)$ -th disclination lines the cell gap is between  $(2N-1)P/4$  and  $(2N+1)P/4$ , where  $P$  is the pitch value. For  $\lambda = 546 \text{ nm}$ ,  $\Delta n = 0.243$  and  $P = 56 \text{ }\mu\text{m}$ , the ratio  $\theta/\Gamma$  is about 0.04, i.e., the twist angle is much smaller than the phase retardation.



**FIGURE 3** The transmittance simulation of the wedge cell (a)  $\psi_0 = 3^\circ$  (b)  $\psi_0 = 45^\circ$ .

We rewrite Eq. (3) by the factor of  $\theta/\Gamma$  and  $(\theta/\Gamma)^2$  which are both much smaller than 1, the equation will be modified as

$$\begin{aligned}
 T &= \frac{1}{2} \left( \frac{\sin^2(2\psi_0) + 4\frac{\theta^2}{\Gamma^2}}{1 + 4\frac{\theta^2}{\Gamma^2}} \right) \left( 1 - \cos \left( \Gamma \sqrt{1 + 4\frac{\theta^2}{\Gamma^2}} \right) \right) \\
 &= \frac{1}{2} \left( \sin^2(2\psi_0) + 4\frac{\theta^2}{\Gamma^2} \right) \left( 1 - \cos \left( \Gamma \left( 1 + 2\frac{\theta^2}{\Gamma^2} \right) \right) \right)
 \end{aligned} \tag{4}$$

This transmittance is a product of two terms. The first term contributes the transmittance amplitude, which increases with the domain order,  $N$ . At  $\psi_0 = 45^\circ$ , this term is close to the constant 1 and the peak of transmittance is almost the same. If  $\psi_0$  is small, however, an abrupt change of this amplitude is resulted at positions of disclination lines. The second term is roughly a cosine function of  $\Gamma$ . The fringe patterns are resulted from this term. Because  $2\theta^2/\Gamma^2$  is much smaller 1, its effect on the number of fringes is negligible.

In Figure 3 we show a simulated result for  $T$  by applying Eq. (4) with  $\lambda = 546$  nm,  $\Delta n = 0.243$  (For E7) and  $P = 56$   $\mu$ m. In Figure 3(a),  $\psi_0 = 3^\circ$  and  $\varphi_0 = 45^\circ$  in Figure 3(b). Although the discontinuities at the disclination lines exist in both cases, the over all intensity difference between domains are much prominent for small  $\psi_0$ . Therefore, it is much easier to distinguish the disclination lines if a small  $\psi_0$  is used. The number of fringes between two adjacent disclination lines keeps the same regardless of the value for angle  $\psi_0$ .

The fringes in each domain is caused by the change of phase retardation  $\Gamma$  within each domain as expressed by the second term in Eq. (4). The change of phase retardation between two adjacent bright (dark) lines is  $2\pi$ , which corresponds to a cell thickness change of  $\lambda/\Delta n$ . On the other hand, the gap difference between two adjacent disclination lines, which can also be clearly distinguished with bare eyes, is  $P/2$ . Therefore, the pitch  $P$  can be determined by simply counting the fringe number  $N_f$  in each domain and using the relation of

$$P = 2N_f\lambda/\Delta n. \quad (5)$$

#### IV. DISCUSSION

In Figure 2 we have shown the photographs of three wedge cells with different spacers. No matter what the spacer thickness is, the numbers of fringes between two adjacent disclination lines are the same. In this method the uniformity of the wedge-shaped is not required. The number of fringes between the adjacent disclination lines is always the same even if they are deformed. Since  $N_f$  can be easily obtained with accuracy of  $\pm 1/4$ , the sensitivity of this new method is  $\pm \lambda/2\Delta n$ . Therefore, this method is good for samples with pitch values larger than a few tens of microns. In the azimuthal anchoring strength measurement, the pitch of chiral doped liquid crystal must be larger than  $4d$ . The sensitivity of pitch measurement in this method is enough for this purpose.

By counting the number of fringes directly in Figure 2, which gives us the number of 12.5, the pitch of the filled chiral nematic liquid crystal is then deduced as 56.17  $\mu$ m by Eq. (5). For our LC (E7 with 0.15%

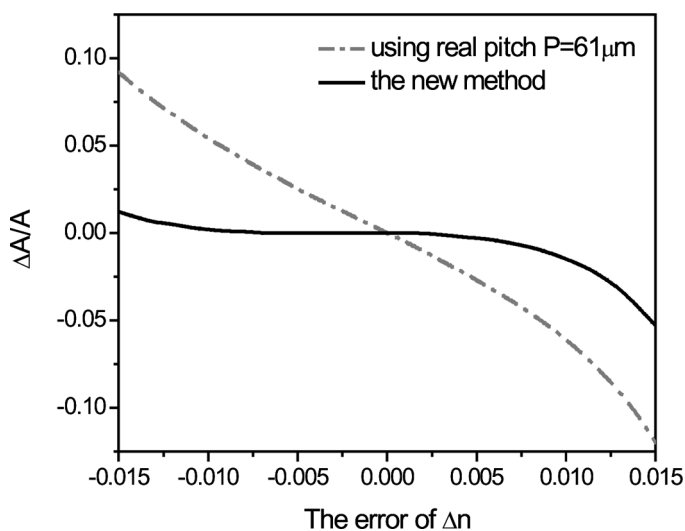


chiral dopant ZLI-811) the calculated pitch value is  $62\mu\text{m}$  at  $20^\circ\text{C}$  by using the twisting power supplied by supplier. We have also used the traditional Grandjean-Cano wedge method and a pitch value of  $65.36\mu\text{m}$  is obtained. The birefringence  $\Delta n$  of the LC is the main reason of forming the fringes and its accuracy is an important factor to get accurate  $P$  value. This seems a drawback of this method; however, we will show in the following paragraphs that this method will give more accurate value for anchoring strength determination.

Since the value  $P$  is to be used to determining the azimuthal energy of a surface to liquid crystal, we now discuss the error may caused by an error of birefringence  $\Delta n$ . The azimuthal anchoring strength can be calculated using the elastic theory and is expressed as [1]

$$A = \frac{2 \times K_{22}}{\sin \theta} \left( \frac{2\pi}{P} - \frac{\theta}{d} \right), \quad (6)$$

where  $\theta$  is the twist angle of nematic in the cell and  $K_{22}$  is the twist elastic constant [3]. Experimentally, we measure the twist angle  $\theta$  by optical method based on the transmittance relation exactly the same as Eq. (1). In other words, we also need  $\Delta n$  in determining the twist angle when measuring the azimuthal anchoring strength. The birefringence usually changes when a nematic liquid crystal is



**FIGURE 4** Analysis on the normalized error of anchoring energy  $\Delta A/A$  versus the error of  $\Delta n$ . The solid line is calculated by using the real pitch value. The dashed line is by using the pitch value measured with this new method.

doped with a chiral dopant. If a same error is introduced both in measuring the pitch and the twist angle, what is the resulted error  $\Delta A$  for the final measurement of anchoring strength? With Figure 4, we have used a hypothetic case of anchoring strength of  $1 \times 10^{-5} \text{ J/m}^2$ , pitch value of  $61 \mu\text{m}$ , thickness of  $11.5 \mu\text{m}$  and  $\Delta n$  of 0.243 to compare the accuracy of the deduced anchoring strength by using the real pitch value and the measured pitch value with error caused by the error of  $\Delta n$ . In Figure 4, we show the normalized error of anchoring strength  $\Delta A/A$  with respect the error in  $\Delta n$  for the two cases: using real pitch value and using the measured pitch value from method described in this work. We see that the error in anchoring strength is always smaller by using the pitch value determined in this method in the reasonable range for  $\Delta n$  is. In this calculation, the error in anchoring strength determination can be decreased from 9% to 1%.

## V. CONCLUSION

This new method for determining the pitch value is simple and accurate. Even if the wedge cell has deformations, the number of fringes between two adjacent disclination lines remains unchanged. By counting the number of the fringes, the pitch value can be determined. The sensitivity of this new method is  $\pm \lambda/2\Delta n$ . Because the principle of measuring the twist angle in determining the azimuthal anchoring strength is the same as that used here, the accuracy of azimuthal anchoring strength is further improved with this new method. Although the birefringence is required for this new method, it introduces an error in pitch determination, however, by applying the pitch value determined this way the deduced azimuthal anchoring strength is more accurate than using a real value.

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